# Analysis of Orthotropic Thin Rectangular Plate with all Edges Clamped Supporting Lateral Loads.

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**Abstract.** The analysis of thin rectangular orthotropic all round clamped plate carrying lateral loads was investigated in this study. The Ritz total potential energy functional was used. The minimization of the total potential energy functional gave the expression for the coefficient of deflection. The coefficient of deflection was used to derive the equation for the lateral load parameter of an orthotropic thin rectangular plate carrying lateral load. A polynomial shape function which was obtained by the direct integration of the governing equation was used to obtain the stiffness coefficients which were substituted in the load parameter equation to obtain the load parameter coefficients for a CCCC plate carrying lateral forces. Numerical examples using permissible deflection (varying from 5mm to 20mm with 5mm interval) and plate thickness (varying from 5mm to 12.5 mm with 0.5mm intervals) were done to determine the load parameters corresponding to an orthotropic thin rectangular CCCC plate carrying lateral loads (when $n_1 = E_y/E_x = 0.7$ and $n_2 = G/E_x = 0.41$) for aspect ratios (b/a) of 1.0, 1.25 and 1.50

**Keywords**: Orthotropic, Plates, Lateral Loads, Direct Variation, Orthogonal Polynomial Shape Function.

**Notations:** a: Length of the plate along x axis, b: Length of the plate along y axis, w: Deflection equation of the plate, A:Coefficient of deflection of the plate, h :Shape function of the plate, $\varepsilon_x$ : normal strain along x – direction, $\varepsilon_y$: normal strain along y – direction, $\gamma_{xy}$: Shear Strain on x – y plane, $\mu_{xy}$: Poisson ratio along x axis, $\mu_{yx}$: Poisson ratio on y axis, $E_x$: Elastic modulus in the x direction, $E_y$: Elastic modulus in the y direction, $G_{xy}$: Shear modulus in the x-y plane, ∝: Aspect Ratio = b/a, t: Thickness of the plate, x: Primary axis of the plate, y:Secondary axis of the plate, z: Axis corresponding to the thickness of the plate, C: Clamped Support, R: Non-dimensional coordinate equal to x/a, Q: Non-dimensional coordinate equal to y/b, q: Lateral load uniformly distributed, $n_1$: Ratio of the young modulus in y direction to the young modulus in the x direction, $n_2$: Ratio of the shear modulus in x-y plane to the young modulus in the x direction.

### 1.0 Introduction

Flat plates are initially flat structural members bounded by two parallel planes, called faces and rectilinear or curvilinear surface called an edge or boundary. The generators of the cylindrical surface are perpendicular to the plane faces. The distance between these plane faces is called the thickness, which is small as compared with the other characteristic dimensions of the plate (Kapadiya H. M. and Patel A. D, 2015). Szilard (2004) defined thin plates as one whose ratio of its basic dimension to its thickness falls within the range $10 \leq a/h \leq 80$.

Plates are used greatly in many fields including but not limited to aerospace, naval, marine, mechanical, architectural, structural, and highway engineering. Specifically, plates are used in bridge decks, naval and marine structures, architectural structures, containers, airplane panels, spacecraft panels, ship decks, machine parts (components) and hydraulic structures. They are classified by their plan shapes as rectangular, circular, elliptical, square, triangular etc. They are also classified according to their materials of construction as homogeneous, heterogeneous, isotropic, anisotropic and orthotropic.

Thin rectangular plates can support loads in many ways. They can be laterally loaded (loads parallel to the thickness), axially or biaxially loaded with in-plane loads (loads perpendicular to the thickness) or a combination of lateral and in-plane load. They can also support dynamic loading. A thin rectangular panel subjected to increasing in-plane compressive load will at a point transit from its stable state of equilibrium to the unstable one, just like columns. Such transition is known as structural instability or buckling (Osadebe et al., 2016). Many aspects of the analysis of thin plates subjected to different loading conditions have been carried out by some scholars. The use of energy methods such as the Ritz method, Galerkin's equilibrium method, work error method was used by Ibearugbulem et al. (2014) to carry out the analysis of thin isotropic plates for the cases of pure bending and also for buckling cases. The flexural analysis of rectangular Kirchhoff plates with clamped and simply supported edges was done by Nwoji et al. (2017). They compared the maximum deflection and maximum bending moments obtained with solutions obtained by Timoshenko and Woinowsky-Krieger, and found them to be in good agreement for a four term displacement shape function. Cui (2007), in his master of philosophy thesis did the exact bending solutions of clamped rectangular thin plates using the symplectic elasticity approach and recorded that the maximum bending moment coefficients obtained were in agreement with the works of Timoshenko and Woinowsky-Krieger. The Galerkin method has been used by Onwuka and Iwuoha (2017), Iwuoha (2016) and Yattender (2005) in the buckling analysis of isotropic thin rectangular plates. While Onwuka and Iwuoha (2017) did on CCCC isotropic plate, Ventsel and Krauthermmer, Iyengar and Chajes, individually carried out works on biaxially loaded SSSS plate subjected to uniform in-plane loads on both axis. Ezeh et al. (2013) used finite difference method for the analysis of laterally loaded thin plates.

**2.0 Theoretical Background**

The Ritz total potential energy functional for pure bending of an isotropic plate is given as:

$$\Pi = \frac{D}{2}\iint\left[\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + \left(\frac{\partial^2 w}{\partial_x \partial_y}\right)^2 + \left(\frac{\partial^2 w}{\partial y^2}\right)^2\right]\partial x\,\partial y - q\iint w\,\partial x\,\partial y \qquad 1$$

Equation 1 is a functional. Minimizing of Equation 1 with respect to the displacement, w gives the equilibrium equation or the resultant force acting on the plate from where the problem can be solved.

**3.0 Methodology**

The method used in this work is as presented below.

**3.1 Formulation of the Equation for the Lateral Load Parameter for an Orthotropic Thin Rectangular Plate carrying Lateral Loads**

The total potential energy functional of an orthotropic thin rectangular plate carrying lateral load is obtained by writing Equation 1 in terms of an orthotropic plate where the plate material properties are not the same in all axes. This was done by Bertram [1] in his master's degree thesis. This is as given in Equation 2.

$$\Pi = \frac{1}{2}\iint\left[D_X\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + 2B\left(\frac{\partial^2 w}{\partial_x \partial_y}\right)^2 + D_Y\left(\frac{\partial^2 w}{\partial y^2}\right)^2\right]\partial x\,\partial y - q\iint w\,\partial x\,\partial y \qquad 2$$

Where,

$$D_X = \frac{E_x t^3}{12(1 - n_1\mu_{xy}^2)} \qquad 3$$

$$D_Y = \frac{E_y t^3}{12(1 - n_1\mu_{xy}^2)} \qquad 4$$

$$B = \frac{n_1 E_x.\mu_{xy}\, t^3}{12\left(1 - n_1\mu_{xy}^2\right)} + \frac{2Gt^3}{12} \qquad 5$$

$n_1$ is the ratio of modulus of elasticity along y axis to that along x axis, $E_y/E_x$.

Consider a thin, rectangular, orthotropic plate, clamped on all edges and supporting uniform distributed lateral load, q as shown in Figure 1.

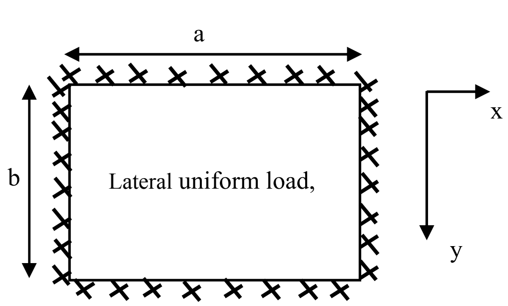


**Figure 1:** Schematic Representation of CCCC plate carrying uniformly distributed lateral load (q)

The Cartesian coordinates, x and y turned to are non-dimensional coordinates, R and Q and expressed as:

$$R = \frac{x}{a}; \qquad Q = \frac{y}{b} \qquad 6$$

The deflection and the slope are zero along the clamped edges. Thus, the boundary conditions of the CCCC plate are

$$w(R = 0) = 0;\ w'^{R}(R = 0) = 0 \qquad 7$$

$$w(R = 1) = 0;\ w'^{R}(R = 1) = 0 \qquad 8$$

$$w(Q = 0) = 0;\ w'^{Q}(Q = 0) = 0 \qquad 9$$

$$w(Q = 1) = 0;\ w'^{Q}(Q = 1) = 0 \qquad 10$$

Where $w'^{R}$ and $w'^{Q}$ are the first derivatives of the displacement functions with respect to R and Q respectively.

From Equation 6, x = Ra and y = Qb.

Substituting x = Ra, y = Qb and $\propto = b/a$ into Equation 2 gives:

$$\Pi = \frac{ab}{2a^4}\int_0^1\int_0^1\left(D_x\left[\frac{\partial^2 w}{\partial R^2}\right]^2 + \frac{2B}{\propto^2}\left[\frac{\partial^2 w}{\partial R\,\partial Q}\right]^2 + \frac{D_y}{\propto^4}\left[\frac{\partial^2 w}{\partial Q^2}\right]^2\right)dR\,dQ - ab\int_0^a\int_0^b (qw)\,dR\,dQ \qquad 11$$

**General Variation of Total Potential Energy Functional**

Minimizing Equation 11 with respect to deflection, w gives the equilibrium equation, otherwise known as the governing equation of equilibrium of forces as:

$$F = \frac{d\Pi}{dw} = \int_0^1\int_0^1\left(D_x\frac{\partial^4 w}{\partial R^4} + \frac{2B}{\propto^2}\frac{\partial^4 w}{\partial R^2\,\partial Q^2} + \frac{D_y}{\propto^4}\frac{\partial^4 w}{\partial Q^4} - qa^4\right)dR\,dQ = 0 \qquad 12$$

The solution of Equation 12 is summarized as shown in Equation 13.

$$w = [1\ R\ R^2 R^3 R^4]\begin{bmatrix} a_0 \\ a_1 \\ a_2/2 \\ a_3/6 \\ a_4/24 \end{bmatrix} \times [1\ Q\ Q^2 Q^3 Q^4]\begin{bmatrix} b_0 \\ b_1 \\ b_2/2 \\ b_3/6 \\ b_4/24 \end{bmatrix} \qquad 13$$

Equation 13 shows the deflection of the plate as a product of the coefficient of deflection (A) and an orthogonal polynomial shape function (h).

$$w = Ah \qquad 14$$

Substituting w = Ah as given in Equation 14 into Equation 11 gives

$$\Pi = \frac{A^2}{2a^4}\iint\left[D_X\left(\frac{\partial^2 h}{\partial R^2}\right)^2 + \frac{2B}{\propto^2}\left(\frac{\partial^2 h}{\partial R\,\partial Q}\right)^2 + \frac{D_Y}{\propto^4}\left(\frac{\partial^2 h}{\partial Q^2}\right)^2\right] ab\,\partial R\,\partial Q - qA\iint hab\,\partial R\,\partial Q \qquad 15$$

Equation 15 can be written as shown in Equation 16

$$\Pi = \frac{A^2}{2a^4}\left[D_X K_X + \frac{2B}{\propto^2}K_{XY} + \frac{D_Y}{\propto^4}K_Y\right] ab - qAK_q ab \qquad 16$$

Where,

$$K_X = \iint\left(\frac{\partial^2 h}{\partial R^2}\right)^2 \partial R\,\partial Q \qquad 17$$

$$K_Y = \iint\left(\frac{\partial^2 h}{\partial Q^2}\right)^2 \partial R\,\partial Q \qquad 18$$

$$K_{XY} = \iint\left(\frac{\partial^2 h}{\partial R\,\partial Q}\right)^2 \partial R\,\partial Q \qquad 19$$

$$K_q = \iint h\,\partial R\,\partial Q \qquad 20$$

**Direct Variation of Total Potential Energy Functional**

If Equation 16 is minimized with respect to the coefficient of deflection, A, direct governing equation (Equation 21) is obtained.

$$\frac{\partial \Pi}{\partial A} = 0 = \frac{A}{a^4}\left[D_X K_X + \frac{2B}{\propto^2}K_{XY} + \frac{D_Y}{\propto^4}K_Y\right] ab - qK_q ab \qquad 21$$

Rearranging Equation 21, gives the coefficient of deflection (A) as:

$$A = \frac{qa^4 K_q}{D_X K_X + \frac{2B}{\propto^2}K_{XY} + \frac{D_Y}{\propto^4}K_Y} \qquad 22$$

Equation 22 can be written as

$$\frac{AD_X}{qa^4} = \frac{[K_q]}{K_X + \frac{2\emptyset_{12}}{\propto^2}K_{XY} + \frac{\emptyset_2}{\propto^4}K_Y} \qquad 23$$

Where,

$$\emptyset_{12} = \frac{B}{D_X} \qquad 24$$

$$\emptyset_2 = \frac{D_Y}{D_X} \qquad 25$$

$$\frac{AD_X}{qa^4} = \frac{K_q}{K_T} \qquad 26$$

Where;

$$K_T = K_X + \frac{2\emptyset_{12}}{\propto^2}K_{XY} + \frac{\emptyset_2}{\propto^4}K_Y \qquad 27$$

Substituting Equation 26 into Equation 14 gives

$$\frac{wD_X}{qa^4} = \frac{K_q}{K_T}h \qquad 28$$

$$\frac{wD_X}{qa^4} = K_m h \qquad 29$$

Where, $K_m$ is given as the ratio of load stiffness coefficient to total material stiffness coefficient. This is defined mathematically as:

$$K_m = \left[\frac{K_q}{K_T}\right] \qquad 30$$

Let $w_{all}$ represent the allowable deflection of the plate. Therefore,

$$w_{all}\left[\frac{D_X}{qa^4}\right] \geq K_m h \qquad 31$$

$$\left[\frac{qa^4}{D_X}\right] \leq \frac{w_{all}}{K_m h} \qquad 32$$

Substituting Equation 3 into Equation 32

$$\left[\frac{12qa^4(1 - n_1\mu_{xy}^2)}{E_x t^3}\right] \leq \frac{w_{all}}{K_m h} \qquad 33$$

Rearranging Equation 33 gives

$$\left[\frac{qa^4}{t^3}\right] \leq \frac{w_{all}\,.\,E_x}{12\,K_m h(1 - n_1\mu_{xy}^2)} \qquad 34$$

$$qa(a/t)^3 \leq \frac{w_{all}\,.\,E_x}{12\,K_m h\left(1 - n_1\mu_{xy}^2\right)} \qquad 35$$

**3.2 Peculiar Deflection Function and Stiffness Coefficients of Orthotropic Thin Rectangular CCCC Plate**

The deflection equation shown in Equation 13 can be written for the x axis as:

$$w_x = a_0 + a_1 R + a_2 R^2 + a_3 R^3 + a_4 R^4 \qquad 36$$

Differentiating Equation 36 once with respect to R, gives Equation 37

$$w_x{}' = a_1 + 2a_2 R + 3a_3 R^2 + 4a_4 R^3 \qquad 37$$

Substituting the boundary conditions given in Equations 7 and 8 into Equations 36 and 37 gives:

$$a_0 = 0, \quad a_1 = 0, \quad a_2 = a_4 \text{ and } a_3 = -2a_4 \qquad 38$$

Substituting Equations 38 into Equation 36 gives the deflection equation for CCCC plate along the x axis as:

$$w_x = a_4(R^2 - 2R^3 + R^4) \qquad 39$$

In a similar manner, the deflection equation for CCCC plate along the y axis is given as:

$$w_y = b_4(Q^2 - 2Q^3 + Q^4) \qquad 40$$

The general deflection equation is, therefore, obtained as the product of the deflection equations in both x and y axes:

$$w = w_x w_y = a_4 b_4(R^2 - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4) \qquad 41$$

Where: $A = a_4 b_4$

$$h = (R^2 - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4) \qquad 42$$

**3.3 Determination of the Maximum Lateral Load Parameter for CCCC orthotropic plate**

The stiffness coefficients given in Equations 17 to 20 are determined using Equation 42 as follows:

$$K_X = \int_0^1\int_0^1\left(\frac{\partial^2 h}{\partial R^2}\right)^2 \partial R\,\partial Q$$
$$= \left(4-\frac{48}{2}+\frac{192}{3}-\frac{288}{4}+\frac{144}{5}\right)\left(\frac{1}{5}-\frac{4}{6}+\frac{6}{7}-\frac{4}{8}+\frac{1}{9}\right) = 0.001269841 \qquad 43$$

$$K_{XY} = \int_0^1\int_0^1\left(\frac{\partial^2 h}{\partial R\,\partial Q}\right)^2 \partial R\,\partial Q$$
$$= \left(\frac{4}{3}-\frac{24}{4}+\frac{52}{5}-\frac{48}{6}+\frac{16}{7}\right)\left(\frac{4}{3}-\frac{24}{4}+\frac{52}{5}-\frac{48}{6}+\frac{16}{7}\right) = 0.00036281179 \qquad 44$$

$$K_Y = \int_0^1\int_0^1\left(\frac{\partial^2 h}{\partial Q^2}\right)^2 \partial R\,\partial Q$$
$$= \left(\frac{1}{5}-\frac{4}{6}+\frac{6}{7}-\frac{4}{8}+\frac{1}{9}\right)\left(4-\frac{48}{2}+\frac{192}{3}-\frac{288}{4}+\frac{144}{5}\right) = 0.001269841 \qquad 45$$

$$K_q = \int_0^1\int_0^1 h\,\partial R\,\partial Q = \left(\frac{1}{3}-\frac{2}{4}+\frac{1}{5}\right)\left(\frac{1}{3}-\frac{2}{4}+\frac{1}{5}\right) = 0.00111111 \qquad 46$$

$h_{max}$ for CCCC occurs at $R = Q = \frac{1}{2}$

$$h_{max} = \left[\left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^4\right]\left[\left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^4\right] = 0.00390625 \qquad 47$$

**3.4 Numerical example**

The material properties of CCCC plate include:

$E_x = 207\times 10^9\ N/m^2$; $\mu_{xy} = 0.3$; $0.1 \le E_y/E_x \le 1$; $0.385 \le G/E_x \le 0.415$; $0.03 \le \mu_{yx}/\mu_{xy} \le 0.3$; $80 \le a/t \le 200$; $w_{all} = 0.005\ m, 0.010\ m, 0.015\ m, 0.020\ m$; Span, $a = 1\ m$

Substituting Equations 43, 44 and 45 into Equation 27 gives

$$K_T = 0.001269841 + \frac{0.000725624\emptyset_{12}}{\alpha^2} + \frac{0.001269841\emptyset_2}{\alpha^4} \qquad 48$$

Substituting Equations 46 and 48 into Equation 30 gives

$$K_m = \frac{0.00111111}{0.001269841 + \frac{0.000725624\emptyset_{12}}{\alpha^2} + \frac{0.001269841\emptyset_2}{\alpha^4}} \qquad 49$$

$$K_m = \frac{1}{1.142858 + \frac{0.65306623\emptyset_{12}}{\alpha^2} + \frac{1.142858\emptyset_2}{\alpha^4}} \qquad 50$$

Substituting Equation 3 and 5 into Equation 24

$$\emptyset_{12} = \left(\frac{n_1 E_x.\mu_{xy}\, t^3}{12(1-n_1\mu_{xy}^2)} + \frac{2Gt^3}{12}\right)\times\frac{12(1-n_1\mu_{xy}^2)}{E_x t^3} \qquad 51$$

Simplifying Equation 51 gives

$$\emptyset_{12} = n_1.\mu_{xy} + 2n_2.(1-n_1\mu_{xy}^2) \qquad 52$$

Substituting Equations 3 and 4 into Equation 25 gives

$$\emptyset_2 = \frac{E_y t^3}{12(1-n_1\mu_{xy}^2)}\times\frac{12(1-n_1\mu_{xy}^2)}{E_x t^3} = \frac{E_y}{E_x} = n_1 \qquad 53$$

If Equations 47 and 50 are substituted into Equation 35 then Equation 54 is obtained:

$$qa(a/t)^3 \le \frac{w_{all}\,.\,E_x\left(1.142858 + \frac{0.65306623\emptyset_{12}}{\alpha^2} + \frac{1.142858\emptyset_2}{\alpha^4}\right)}{12\ x\ 0.00390625(1-n_1\mu_{xy}^2)} \qquad 54$$

$$qa(a/t)^3 \le \frac{w_{all}\,.\,E_x\left(24.380971 + \frac{13.9319996\emptyset_{12}}{\alpha^2} + \frac{24.380971 n_1}{\alpha^4}\right)}{(1-n_1\mu_{xy}^2)} \qquad 55$$

**4.0 Results and Discussion**

The values of the parameter $\phi_{12}$ required for the calculation of the lateral load parameter for an orthotropic plate are presented on Table 1. Plate thickness varying from 5mm to 12.5mm (with 0.5mm intervals), allowable deflection varying from 5mm to 20mm (with 5mm interval), aspect ratio values varying from 1.0 to 2.25 (with 0.25 interval) were used for the computation of the Load Parameters and the results presented on Table 2. The results presented on Table 2 were plotted for a square plate as shown on Figure 1. From Figure 1 and Table 2, it could be observed that, as the value of plate thickness increases, the Load Parameter of CCCC plate increases for any allowable deflection. This is so because the plate tends to sustain more loads when the thickness is increased due to increase in the stiffness of the plate.

The maximum deflection coefficients for the CCCC plate were calculated using Equation 32, in order to validate the results of this work. The results obtained at $n_1$ =1 (isotropic case) were compared with that of Ibearugbulem et al. (2014) as presented on Table 3. From Table 3, it is seen that, the results obtained from this present work when $n_1$ =1, is in very much agreement with established results of laterally loaded isotropic CCCC plate for different aspect ratios as the maximum percentage difference is -0.340. This therefore, validates the results of the generated load coefficients.

**Table 1:** Values of the parameter $\phi_{12}$ required for the calculation of the lateral load parameter for an orthotropic plate

| | $\phi_{12}$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | $n_2$ | | | | | | |
| | | **0.385** | **0.39** | **0.395** | **0.4** | **0.405** | **0.41** | **0.415** |
| $n_1$ | **0.1** | 0.79307 | 0.80298 | 0.81289 | 0.8228 | 0.83271 | 0.84262 | 0.85253 |
| | **0.2** | 0.81614 | 0.82596 | 0.83578 | 0.8456 | 0.85542 | 0.86524 | 0.87506 |
| | **0.3** | 0.83921 | 0.84894 | 0.85867 | 0.8684 | 0.87813 | 0.88786 | 0.89759 |
| | **0.4** | 0.86228 | 0.87192 | 0.88156 | 0.8912 | 0.90084 | 0.91048 | 0.92012 |
| | **0.5** | 0.88535 | 0.89490 | 0.90445 | 0.9140 | 0.92355 | 0.93310 | 0.94265 |
| | **0.6** | 0.90842 | 0.91788 | 0.92734 | 0.9368 | 0.94626 | 0.95572 | 0.96518 |
| | **0.7** | 0.93149 | 0.94086 | 0.95023 | 0.9596 | 0.96897 | 0.97834 | 0.98771 |
| | **0.8** | 0.95456 | 0.96384 | 0.97312 | 0.9824 | 0.99168 | 1.00096 | 1.01024 |
| | **0.9** | 0.97763 | 0.98682 | 0.99601 | 1.0052 | 1.01439 | 1.02358 | 1.03277 |
| | **1.0** | 1.00070 | 1.00980 | 1.01890 | 1.0280 | 1.03710 | 1.04620 | 1.05530 |

**Table 2a:** Load parameter (q.a) of given plate thickness and permissible deflection for thin orthotropic CCCC plate in bending (when $n_1 = E_y/E_x = 0.7$ and $n_2 = G/E_x = 0.41$) for aspect ratios (b/a) of 1.0, 1.25 and 1.50

| t (mm) | q.a | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | b/a = 1 | | | | b/a = 1.25 | | | | b/a = 1.5 | | | |
| | $w_{all}$ = 5 mm | $w_{all}$ = 10 mm | $w_{all}$ = 15 mm | $w_{all}$ = 20 mm | $w_{all}$ = 5 mm | $w_{all}$ = 10 mm | $w_{all}$ = 15 mm | $w_{all}$ = 20 mm | $w_{all}$ = 5 mm | $w_{all}$ = 10 mm | $w_{all}$ = 15 mm | $w_{all}$ = 20 mm |
| 5 | 7.6048 | 15.2096 | 22.8144 | 30.4192 | 5.5360 | 11.0721 | 16.6081 | 22.1442 | 4.6683 | 9.3366 | 14.0048 | 18.6731 |
| 5.5 | 10.1220 | 20.2440 | 30.3660 | 40.4880 | 7.3685 | 14.7369 | 22.1054 | 29.4739 | 6.2135 | 12.4270 | 18.6404 | 24.8539 |
| 6 | 13.1411 | 26.2822 | 39.4233 | 52.5644 | 9.5663 | 19.1326 | 28.6988 | 38.2651 | 8.0668 | 16.1336 | 24.2003 | 32.2671 |
| 6.5 | 16.7078 | 33.4155 | 50.1233 | 66.8310 | 12.1627 | 24.3254 | 36.4880 | 48.6507 | 10.2562 | 20.5124 | 30.7686 | 41.0248 |
| 7 | 20.8676 | 41.7352 | 62.6027 | 83.4703 | 15.1909 | 30.3818 | 45.5727 | 60.7636 | 12.8097 | 25.6195 | 38.4292 | 51.2390 |
| 7.5 | 25.6662 | 51.3324 | 76.9986 | 102.6648 | 18.6841 | 37.3683 | 56.0524 | 74.7365 | 15.7554 | 31.5109 | 47.2663 | 63.0217 |
| 8 | 31.1493 | 62.2985 | 93.4478 | 124.5971 | 22.6756 | 45.3512 | 68.0268 | 90.7025 | 19.1213 | 38.2425 | 57.3638 | 76.4850 |
| 8.5 | 37.3624 | 74.7248 | 112.0872 | 149.4496 | 27.1986 | 54.3971 | 81.5957 | 108.7942 | 22.9352 | 45.8705 | 68.8057 | 91.7410 |
| 9 | 44.3512 | 88.7024 | 133.0536 | 177.4049 | 32.2862 | 64.5724 | 96.8585 | 129.1447 | 27.2254 | 54.4508 | 81.6762 | 108.901 |
| 9.5 | 52.1613 | 104.322 | 156.4840 | 208.6454 | 37.9717 | 75.9434 | 113.915 | 151.8868 | 32.0197 | 64.0394 | 96.0591 | 128.078 |
| 10 | 60.8384 | 121.676 | 182.5153 | 243.3537 | 44.2883 | 88.5766 | 132.864 | 177.1532 | 37.3462 | 74.6924 | 112.038 | 149.384 |
| 10.5 | 70.4281 | 140.856 | 211.2842 | 281.7123 | 51.2693 | 102.538 | 153.807 | 205.0770 | 43.2329 | 86.4658 | 129.698 | 172.931 |
| 11 | 80.9759 | 161.951 | 242.9278 | 323.9038 | 58.9477 | 117.895 | 176.843 | 235.7910 | 49.7078 | 99.4156 | 149.123 | 198.831 |
| 11.5 | 92.5276 | 185.055 | 277.5829 | 370.1106 | 67.3570 | 134.714 | 202.071 | 269.4279 | 56.7989 | 113.597 | 170.396 | 227.195 |
| 12 | 105.128 | 210.257 | 315.3864 | 420.5152 | 76.5302 | 153.060 | 229.590 | 306.1208 | 64.5342 | 129.068 | 193.602 | 258.137 |
| 12.5 | 118.825 | 237.650 | 356.4752 | 475.3002 | 86.5006 | 173.001 | 259.501 | 346.0024 | 72.9418 | 145.883 | 218.825 | 291.767 |

**Table 2b:** Load parameter (q.a) of given plate thickness and permissible deflection for thin orthotropic CCCC plate in bending (when $n_1 = E_y/E_x = 0.7$ and $n_2 = G/E_x = 0.41$) for aspect ratios (b/a) of 1.0, 1.25 and 1.50

| t (mm) | q.a | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | b/a = 1.75 | | | | b/a = 2.0 | | | | b/a = 2.25 | | | |
| | $w_{all}$ = 5 mm | $w_{all}$ = 10 mm | $w_{all}$ = 15 mm | $w_{all}$ = 20 mm | $w_{all}$ = 5 mm | $w_{all}$ = 10 mm | $w_{all}$ = 15 mm | $w_{all}$ = 20 mm | $w_{all}$ = 5 mm | $w_{all}$ = 10 mm | $w_{all}$ = 15 mm | $w_{all}$ = 20 mm |
| 5 | 4.2321 | 8.4643 | 12.6964 | 16.9286 | 3.9841 | 7.9683 | 11.9524 | 15.9366 | 3.8301 | 7.6601 | 11.4902 | 15.3203 |
| 5.5 | 5.6330 | 11.2660 | 16.8989 | 22.5319 | 5.3029 | 10.6058 | 15.9087 | 21.2116 | 5.0978 | 10.1956 | 15.2934 | 20.3913 |
| 6 | 7.3131 | 14.6263 | 21.9394 | 29.2526 | 6.8846 | 13.7692 | 20.6538 | 27.5384 | 6.6183 | 13.2367 | 19.8550 | 26.4734 |
| 6.5 | 9.2980 | 18.5960 | 27.8941 | 37.1921 | 8.7532 | 17.5063 | 26.2595 | 35.0126 | 8.4146 | 16.8293 | 25.2439 | 33.6586 |
| 7 | 11.6130 | 23.2260 | 34.8390 | 46.4520 | 10.9325 | 21.8650 | 32.7975 | 43.7299 | 10.5097 | 21.0194 | 31.5291 | 42.0388 |
| 7.5 | 14.2835 | 28.5670 | 42.8504 | 57.1339 | 13.4465 | 26.8930 | 40.3394 | 53.7859 | 12.9265 | 25.8529 | 38.7794 | 51.7058 |
| 8 | 17.3349 | 34.6697 | 52.0046 | 69.3394 | 16.3190 | 32.6381 | 48.9571 | 65.2762 | 15.6879 | 31.3759 | 47.0638 | 62.7518 |
| 8.5 | 20.7925 | 41.5850 | 62.3776 | 83.1701 | 19.5741 | 39.1482 | 58.7223 | 78.2964 | 18.8171 | 37.6342 | 56.4513 | 75.2684 |
| 9 | 24.6819 | 49.3637 | 74.0456 | 98.7274 | 23.2355 | 46.4710 | 69.7065 | 92.9421 | 22.3369 | 44.6739 | 67.0108 | 89.3477 |
| 9.5 | 29.0283 | 58.0565 | 87.0848 | 116.1131 | 27.3272 | 54.6545 | 81.9817 | 109.3089 | 26.2704 | 52.5408 | 78.8112 | 105.081 |
| 10 | 33.8571 | 67.7143 | 101.5714 | 135.4286 | 31.8731 | 63.7463 | 95.6194 | 127.4925 | 30.6405 | 61.2810 | 91.9215 | 122.562 |
| 10.5 | 39.1939 | 78.3877 | 117.5816 | 156.7755 | 36.8971 | 73.7943 | 110.691 | 147.5886 | 35.4702 | 70.9404 | 106.410 | 141.880 |
| 11 | 45.0639 | 90.1277 | 135.1916 | 180.2554 | 42.4231 | 84.8463 | 127.269 | 169.6926 | 40.7825 | 81.5650 | 122.347 | 163.130 |
| 11.5 | 51.4925 | 102.985 | 154.4774 | 205.9699 | 48.4751 | 96.9501 | 145.425 | 193.9002 | 46.6004 | 93.2008 | 139.801 | 186.401 |
| 12 | 58.5051 | 117.010 | 175.5154 | 234.0206 | 55.0768 | 110.153 | 165.230 | 220.3071 | 52.9468 | 105.893 | 158.840 | 211.787 |
| 12.5 | 66.1272 | 132.254 | 198.3817 | 264.5089 | 62.2522 | 124.504 | 186.756 | 249.0089 | 59.8447 | 119.689 | 179.534 | 239.378 |

**Table 3:** Maximum Deflection coefficients at $n_2 = 1$ (isotropic case).

| Aspect ratio (b/a) | Ibearugbulem et al. (2014) | Present study | Percentage difference |
|---|---|---|---|
| 1.0 | 0.00133 | 0.00132921 | -0.059 |
| 1.1 | 0.00159 | 0.00158587 | -0.260 |
| 1.2 | 0.00182 | 0.00181896 | -0.057 |
| 1.3 | 0.00203 | 0.00202456 | -0.269 |
| 1.4 | 0.00221 | 0.00220251 | -0.340 |
| 1.5 | 0.00236 | 0.00235478 | -0.222 |
| 1.6 | 0.00249 | 0.00248434 | -0.228 |
| 1.7 | 0.00260 | 0.00259437 | -0.217 |
| 1.8 | 0.00269 | 0.00268787 | -0.079 |
| 1.9 | 0.00277 | 0.00276753 | -0.089 |
| 2.0 | 0.00284 | 0.00283565 | -0.153 |



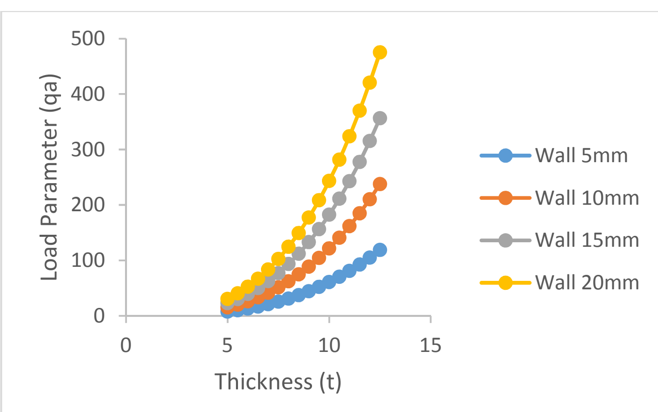


**Figure 1:** Graph of Load parameter coefficients against thickness for a square plate

**Conclusion**

The conclusions drawn from the study are as follows:

The Load parameter equation for an orthotropic thin rectangular plate carrying lateral loads has been determined. The Load parameter coefficients for an orthotropic thin rectangular all-round clamped plate supporting lateral loads, have been determined for different aspect ratios, allowable deflection values and plate thicknesses.

Since the results of the maximum deflection coefficients obtained from this work at $n_1$ = (isotropic case) agrees with the results obtained by Ibearugbulem et al. (2014), it therefore follows that, the results obtained in this work for the Load coefficients at other $n_1$ values (for which there are no existing results in literature), are also correct.

The use of the equations and tables developed in this study is recommended for an easy and quick analysis of the problems of orthotropic thin rectangular plates carrying Lateral load.

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